On Capturing Oil Rents with a National Excise Tax Revisited\textsuperscript{*}

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29 July 2004

\textsuperscript{*}This paper was presented at the I Conference of the Spanish-Portuguese Association of Environmental and Resource Economics, Vigo, 18-19 June 2004 and at the VI Spanish Meeting on Game theory and Practice, Elche, 12-14 July 2004.

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Abstract

In this paper the scope of Bergstrom’s (1982) results is studied. Moreover, his analysis is extended assuming that extraction cost is directly related to accumulated extractions. For the case of a competitive market it is found that the optimal policy is a constant tariff if extraction is costless. However, with depletion effects, the optimal tariff must ultimately be decreasing. For the case of a monopolistic market the results depend crucially on the kind of strategies the importing country governments can play and on whether the monopolist chooses the price or extraction rate. For a price-setting monopolist it is shown that the importing countries cannot use a tariff to capture monopoly rents if they are constrained to use open-loop strategies, even if the governments sign a tariff agreement. This result is drastically modified if the importing countries in the tariff agreement use Markov (feedback) strategies. For a quantity-setting monopolist the nature of the game changes and the importing country governments find it advantageous to set a tariff on resource importations. Moreover, in this case the importing countries in a tariff agreement enjoy a strategic advantage which allows them to behave as a leader.

Keywords: tariffs, tariff agreements, nonrenewable resources, depletion effects, price-setting monopolist, quantity-setting monopolist, differential games, open-loop strategies, linear strategies, Markov-perfect Nash equilibrium, Markov-perfect Stackelberg equilibrium.

JEL Classification System: C73, D41, D42, F02, H20, Q38
1 Introduction

The issue of using an import tariff to capture nonrenewable resource rents was addressed sometime ago in a nice piece of work by Bergstrom (1982). In his paper, he shows that if all importing countries of a competitively supplied nonrenewable resource select the same *ad valorem* tariff on the resource consumed at any time, the tariff is advantageous for the importing countries in the sense that they capture resource rents from the exporting countries. He characterizes the Nash equilibrium of the game among the importing countries by a simple rule relating the equilibrium *ad valorem* tariff to demand elasticities and market shares. In the second part of the paper, he argues that almost all the profits of a monopolist can be taxed away by the importing countries if they choose a sufficiently high tariff as long as the profit-maximizing level of the monopolist’s price is independent of the tariff. These results are obtained for a Hotelling-type model with a costlessly extracted nonrenewable resource. Later, several papers addressed this issue, among them Brander and Djajic (1983), Karp (1984), Maskin and Newbery (1990), Karp and Newbery (1991, 1992). However only a few authors have used for their analysis the game among the importing countries proposed by Bergstrom. In particular, Karp and Newbery (1991) have characterized the Markov equilibria of two games in which large importers who behave strategically confront competitive suppliers of a nonrenewable resource and the marginal extraction cost is positive but constant. For the game where importers move first, they integrate numerically the pair of ordinary differential equations that characterize the perfect, importers move first equilibrium. For the other game, where exporters move first, as the importers have no influence on the dynamics of the resource, the importers choose at each moment a tariff to maximize the instantaneous domestic welfare taking as given the extraction rate and the rival’s tariffs. The result is that the tariff is given by the Nash equilibrium of a static game among the importers.

In this paper we study the scope of the results obtained by Bergstrom using the theory of differential games. Moreover, we extend his analysis by assuming that extraction cost is directly related to accumulated extractions (depletion effects). When large importers confront competitive suppliers we show that the open-loop Nash equilibrium tariff is *constant* only if the marginal extraction cost is zero, which is Bergstrom’s conclusion. We

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1A recent contribution by Hörner and Kamien (2004) shows that the intertemporal no-arbitrage condition that arises if the durable good monopolist seller can commit to a price path mirrors the intertemporal no-arbitrage condition if the monopsonist buyer of an exhaustible resource supplied by competitive sellers can commit to a profile of import tariffs.
also find that if the marginal extraction cost is positive but constant then the optimal tariff is *increasing*. Finally, with depletion effects the optimal tariff must ultimately be *decreasing*. This is an immediate consequence of the fact that the long-run value of the tariff rate is zero because of the economic exhaustion of the resource, which requires that in the long run the marginal extraction cost be equal to the backstop price. This means that the resource rent vanishes in the long run and no tariff can be applied. Our approach of the intertemporal competitive analysis is very similar to the one proposed in the second part of Karp and Newbery’s (1991) paper where the importers move first and the game amongst importers is truly dynamic, although we think that as the consumers in the importing countries and the producers in the exporting countries act *parametrically* with respect to the price, it is not necessary to assume that the importers move first to get a differential game amongst the governments of the importing countries. As we show, the application of the market equilibrium condition at each point in time is enough to yield a differential game amongst the importing country governments. In other words, as consumers and producers of the resource take the price as given there is no strategic interdependence amongst them, so that only when the equilibrium price clears the market does a strategic interdependence appear amongst the governments of the importing countries through their influence on the market equilibrium and hence on the resource dynamics, i.e., the only game that can appear in an international competitive resource market with large importers, without the intervention of the governments of the exporting countries, is amongst the governments of the importing countries.

In the second part of this paper we study the monopoly case using a differential game among $n$ importing countries and the monopolist. Contrary to the results obtained by Bergstrom, we find that the profit-maximizing level of the monopolist’s price is not independent of the tariff. However, our results establish that importing countries cannot use the tariff to capture part of the monopoly’s rents, i.e., that the open-loop Nash equilibrium tariff is zero. This result appears because the user cost for the importing countries is equal to the tariff times the monopoly price and because the extraction cost is supported by the monopolist. In this case the user cost for importing countries must increase at a rate equal to the interest rate but this is incompatible with the fact that ultimately the rent must be zero because of the economic exhaustion of the resource. We also find that this result applies for a *per unit* tariff and when the importing countries cooperate imposing the *same* tariff rate on the resource importations, i.e., when the importing country governments sign a *tariff agreement*, and independently of whether it is assumed that the countries are symmetric or not.
In order to clarify whether this result is a consequence of the equilibrium concept used to solve the game, we propose a differential game between a monopolist and a coalition of importing country governments for which it is possible to calculate the stationary linear Markov strategies. The solution to this game establishes that the importing countries can capture part of the monopoly’s rents for their consumer using a tariff on the resource importations. Thus, we find that when the monopolist chooses a pricing policy the tariff is advantageous for the importing countries if the importing countries co-operate through a tariff agreement and the optimal policy is defined using a feedback strategy.

Finally, we examine the case of a quantity-setting monopolist. For this case, as the importing countries have no influence on the dynamics of the stock, the game among the importing countries becomes, in fact, a static game and given the influence of the tariff rate on the monopoly price the importing countries find it advantageous to set a tariff on the resource importations.\footnote{This is the same result as the one obtained by Karp and Newbery (1991) when competitive suppliers move first in the game. However, we get it here as a consequence of the fact that the monopolist uses the extraction rate as a control variable.} On the other hand, if the governments of the importing countries sign a tariff agreement it is pretty obvious that the open-loop Nash equilibrium policy is to choose, for a given extraction rate, a tariff such that the monopolist’s price be zero, but then the monopoly is not interested in exploiting the resource so that finally the consumers of the importing countries are not going to enjoy any surplus. However, the coalition has another alternative since the importing country governments, in fact, enjoy a strategic advantage given that they can influence the monopoly price through the tariff. In other words, the importing country governments have another alternative because the coalition can behave as a leader. To conclude the analysis we calculate the stationary Markov-perfect Stackelberg equilibrium in linear strategies which guarantees the strong time consistency of the tariff, and we obtain that the importing countries are interested in setting a tariff that provides the monopoly with the possibility of obtaining a positive price with the aim of getting a positive surplus for their consumers. A policy that clearly is superior to the one derived from the open-loop Nash equilibrium.\footnote{This kind of differential game between a quantity-setting monopolist (the follower) and a tariff-setting buyer (the leader) has been studied by Karp (1984). In that paper, he shows that the open-loop Stackelberg tariff is inconsistent because of the stock-dependence of extraction costs. Besides, he proposes an alternative method to the one used in this paper, see Prop. 2 on page 87, of obtaining temporally consistent strategies.}

The paper is organized as follows. In the next section, Section 2, the intertemporal competitive analysis is developed. In Section 3 the case of a
price-setting monopolist is studied and the case of a quantity-setting monopolist is dealt with in Section 4. The comparison between the Markov-perfect Nash equilibrium of the differential game where the monopolist sets up the price and the Markov-perfect Stackelberg equilibrium where the monopolist sets up the extraction rate appears in Section 5. Conclusions and subjects for future research are presented in Section 6.

2 Intertemporal Competitive Analysis

As in Bergstrom’s (1982) analysis, we shall confine ourselves to a partial equilibrium model. Assuming that the representative consumer of the importing country acts as a price-taker agent, we can write the consumer’s welfare function as

\[ U_i(q_i) - p(1 + \theta_i)q_i + R_i, \]

where \( U_i(q_i) \) is the consumer’s gross surplus, \( q_i \) the amount of the resource bought by the representative consumer of the importing country \( i \), \( p \) the international price of the resource, \( \theta_i \) the “ad valorem” tariff rate on the resource imports fixed by the government of the importing country \( i \), and \( R_i \) a lump-sum transfer that the consumer receives from the government. Thus, the resource demand depends only on the consumer price: \( U_i'(q_i) = p(1 + \theta_i) \) and the demand function can be written as

\[ q_i = D_i(p(1 + \theta_i)) \]

with \( D_i' < 0 \) if the marginal utility is decreasing.\(^4\) Thus the aggregate demand is \( Q^D = \sum_{i=1}^{n} D_i(p(1 + \theta_i)). \)

On the other side of the market we have \( m \) identical competitive exporting countries extracting the resource at an aggregate cost equal to \( c(x)Q \), where \( c(x) \) is the marginal extraction cost, with \( c' > 0 \) and \( c'' \geq 0 \), \( x \) stands for the accumulated extractions and \( Q \) for the current extraction rate of the resource. The competitive supply is given by the solution to the following optimal control problem

\[
\max_{\{Q^S\}} \int_0^\infty e^{-rt} \left( (p - c(x))Q^S \right) dt,
\]

s.t. \( \dot{x} = Q^S, \ x(0) = x_0 \geq 0, \)

where \( r \) is the market rate of interest and \( Q^S \) the aggregate supply of the resource. The solution of this model is well known, and yields a modified version of Hotelling’s rule:

\[
\frac{(d/dt)(p - c) + c'Q^S}{p - c} = r,
\]

\(^4\)\( D_i' \) stands for the derivative of the demanded quantity with respect to the consumer price: \( p(1 + \theta_i). \)
i.e., the total return associated with holding a unit of stock must equal the market rate of interest. With depletion effects the total return includes the capital gains plus the reduction in the total extraction cost resulting from that unit. This equation can be written in terms of the price change rate

$$\dot{p} = r(p - c(x)),$$

so that the competitive supply is characterized by the following system of differential equations

$$\dot{x} = Q^S, \quad \dot{p} = r(p - c(x)).$$

In order to close the model the market clearing condition must be imposed, which requires that the demand be equal to the supply at each moment. This condition yields the following differential equation

$$\dot{x} = Q^S = Q^D = \sum_{i=1}^{n} D_i(p^*(1 + \theta_i)),$$

where $p^*$ is the competitive equilibrium price in the world market. This equation along with (1) show that the competitive equilibrium dynamics depends on the tariff imposed by the importing countries. This is a consequence of the fact that both the consumers in the importing countries and the producers in the exporting countries act parametrically, i.e., they take prices as given and choose quantities, the demand and the supply of the resource. So, as the demand depends on the tariff rate set by the governments of the importing countries, finally the market equilibrium condition at each point in time yields a dynamic game among the importing country governments. In this context, if the governments set the tariff with the aim of maximizing the discounted present value of the representative consumer’s welfare ($= \text{consumer surplus} + \text{tariff revenues}$), the optimal time path for the tariff is given by the solution of the following differential game amongst the governments of the $n$ importing countries\(^5\)

$$\max_{\{\theta_i\}} \int_{0}^{\infty} e^{-rt} \left( U_i(D_i(p(1 + \theta_i))) - pD_i(p(1 + \theta_i)) \right) dt,$$

\(^5\)It is assumed that the tariff revenues are reimbursed to the consumers as lump-sum transfers, i.e., that $p\theta_iq_i = R_i$. In order to simplify the notation we omit the asterisk over the price. It should be understood that $p$ in the optimal control problem is the competitive equilibrium international price.
\[ s.t. \quad \dot{x} = Q = \sum_{j=1}^{n} D_j(p(1 + \theta_j)), \quad x(0) = x_0 \geq 0, \]
\[ \dot{p} = r(p - c(x)). \]

### 2.1 The Open-loop Nash Equilibrium

In this section the differential game is solved using open-loop strategies. Although open-loop strategies do not guarantee the subgame perfection (strong time consistency), they are easy to compute and allow us to obtain some analytical results about the dynamics of the tariff with depletion effects.

First, we define the Hamiltonian associated to the optimal control problems presented in the previous section.

\[ H_i = U_i(D_i(p(1 + \theta_i))) - pD_i(p(1 + \theta_i)) + \eta_{i1} \sum_{j=1}^{n} D_j(p(1 + \theta_j)) + \eta_{i2} r(p - c(x)), \]

which yields the following necessary conditions

\[ \frac{\partial H_i}{\partial \theta_i} = pD'_i(U'_i - p + \eta_{i1}) = 0, \quad (2) \]
\[ \dot{\eta}_{i1} = r(\eta_{i1} + \eta_{i2}c'), \quad (3) \]
\[ \dot{\eta}_{i2} = -(1 + \theta_i)U'_iD'_i + D_i + p(1 + \theta_i)D'_i - \eta_{i1} \sum_{j=1}^{n} (1 + \theta_j)D'_j. \quad (4) \]

where (2) and (4) can be simplified using \( U'_i = p(1 + \theta_i) \), resulting in

\[ p \theta_i = -\eta_{i1}, \quad (5) \]
\[ \dot{\eta}_{i2} = -p\theta_i(1 + \theta_i)D'_i + D_i - \eta_{i1} \sum_{j=1}^{n} (1 + \theta_j)D'_j. \quad (6) \]

(5) defines the standard condition which establishes that the optimal tariff must be equal to the user cost of the resource.\(^6\)

The dynamics of the tariff rate can be obtained using (1), (3) and (5).

\[ \dot{\theta}_i = \frac{r}{p}(c\theta_i - \eta_{i2}c'), \]

so that the following result can be established

\(^6\)See (7) in Maskin and Newbery (1990).
**Proposition 1** When the marginal extraction cost is constant, i.e., without depletion effects $c' = 0$, the open-loop Nash equilibrium tariff rate is increasing and its rate of growth is $\dot{\theta}_i/\theta_i = rc/p$.

Moreover, we have

**Corollary 1** (Bergstrom (1982)) If $c = 0$, i.e., when extraction is costless, the open-loop Nash equilibrium tariff rate is constant.

This conclusion provides Bergstrom’s result with generality. In his paper it is written that: “The optimal tax rate found in equation (32) has only been shown to be the optimal constant tax rate. More generally we could allow the possibility that a country could choose a schedule of changing tax rates over time” (Bergstrom (1982, p. 198)). Here we allow for this possibility since we do not impose any *a priori* dynamics for the tax rate as in Bergstrom’s paper and we find that the optimal open-loop tariff rate must be constant.\footnote{In Bergstrom’s analysis it is assumed that: i) A constant excise tax rate is imposed by each country in all periods; ii) All importing countries impose the same excise tax rate on the resource consumed at any time. See again Bergstrom (1982, pp. 196-7). In Kemp and Long (1980), it is also shown that the open-loop tariff must be constant when extraction is costless and increasing when the marginal extraction cost is constant for the case of a monopsonistic importing country.}

### 2.2 The Optimal Open-loop Tariff with Depletion Effects

With depletion effects a system of $3n+2$ equations, (3)-(5)-(6) for $i = 1, ..., n$ and the two dynamic constraints, must be analyzed. Since this is a rather difficult task to tackle we propose a simplification assuming that all the representative consumers of the different importing countries are identical. Thus, next, we focus on a symmetric equilibrium. Assuming that all the consumers are identical and that $\theta = \theta_1 = ... = \theta_n$ the open-loop Nash equilibrium is given by the following system of differential equations:

\[
\begin{align*}
\dot{x} &= nD(p(1 + \theta)), \\
\dot{p} &= r(p - c(x)), \\
\dot{\theta} &= \frac{r}{p}(c(x)\theta_i - \eta_2 c'(x)), \\
\dot{\eta}_2 &= D + (n - 1)p\theta(1 + \theta)D',
\end{align*}
\]

where $p\theta = -\eta_1$ has been used to simplify (6).
The steady state of this system is easily calculated, resulting in \( \theta^\infty = \eta^2_2 = 0 \), \( p^\infty = c(x^\infty) \) and \( D(p^\infty) = 0 \). Using these values we can evaluate the Jacobian matrix of the system (7)-(8)-(9)-(10) at the steady state and establish its stability properties.

\[
J = \begin{bmatrix}
0 & nD' & np^\infty D' & 0 \\
-rc' & r & 0 & 0 \\
0 & 0 & \frac{rc}{p^\infty} & -\frac{rc'}{p^\infty} \\
0 & D' & np^\infty D' & 0
\end{bmatrix}.
\] (11)

Then the characteristic roots \( \rho_k \) (\( k = 1, \ldots, 4 \)) are the solutions of the characteristic equation

\[
\rho^4 - (tr \ J) \rho^3 + \Psi \rho^2 - \Pi \rho + |J| = 0,
\]

where

\[
tr \ J = \frac{r(p^\infty + c)}{p^\infty} > 0,
\]

\[
\Psi = \frac{r \left( \frac{rc}{p^\infty} + 2nc'D' \right)}{p^\infty},
\]

\[
\Pi = \frac{n(p^\infty + c)r^2 D'}{p^\infty} < 0,
\]

\[
|J| = n(n - 1)r^2c^2D'^2 > 0,
\]

\( \Psi \) and \( \Pi \) being the sum of all diagonal second and third order minors of \( J \). According to Dockner (1985, Th. 1) the four roots are

\[
\rho_1, \rho_2, \rho_3, \rho_4 = \frac{r}{2} \pm \frac{\Omega^{1/2}}{2},
\]

(12)

where

\[
\Omega = \Psi - r^2 = -\frac{r^2}{4} - \frac{\Omega}{2} + \frac{1}{2} \left( \Omega^2 - 4 |J| \right)^{1/2},
\]

\[
= 2nc'D' < 0,
\]

since \( p^\infty = c(x^\infty) \) at the steady state, and

---

\(^8\)Notice that the condition \( D' = D = 0 \) would leave the steady-state values of \( p \) and \( \theta \) undetermined. \( D(p(1 + \theta)) = 0 \) yields \( p = \bar{p}/(1 + \theta) \) where \( \bar{p} \) is the intersection point of the demand function with the price axis (backstop price). Then by substitution in \( D' \) we would obtain \( D'(\bar{p}) = 0 \).
$\Omega^2 - 4|J| = 4nr^2c^2D^2 > 0.$

Then we have that the eigenvalues of $J$ are real, two being positive and two being negative (Dockner (1985, Th. 3) and that the sufficient conditions for the (local) saddle point property are satisfied (Tahvonen and Kuuluvainen (1993, Lemma 1)), this means that the system (7)-(8)-(9)-(10) has a unique steady state that is a saddle point.

This stability analysis allows us to conclude that

**Proposition 2** The open-loop Nash equilibrium tariff rate with depletion effects is ultimately decreasing.

This is an immediate consequence of the fact that the steady-state value of the tariff rate is zero. With a steady-state value equal to zero the tariff must be decreasing when it is approaching the long-run equilibrium.

### 3 The Case of a Price-setting Monopolist

When the governments of the importing countries face a monopoly the nature of the game changes. Now the game has another player, the monopolist, who chooses a pricing policy over time so as to maximize the present value of his profits

$$\max_{\{p\}} \int_0^\infty e^{-rt} \left( (p - c(x)) \sum_{i=1}^n D_i(p(1 + \theta_i)) \right) \, dt,$$

$$s.t. \dot{x} = Q = \sum_{i=1}^n D_i(p(1 + \theta_i)), \quad x(0) = x_0 \geq 0.$$

On the other hand, the governments of the importing countries fix, as before, a tariff in order to maximize the discounted present value of the representative consumer’s welfare. Now both types of players face the same dynamic constraint and the optimal time path for the tariff is given by the solution of a *differential game* between the monopolist and the $n$ importing countries.
3.1 The Open-loop Nash Equilibrium

First, we write the Hamiltonian associated to the optimal control problems of the importing countries.

\[ H_i = U_i(D_i(p(1 + \theta_i))) - pD_i(p(1 + \theta_i)) + \lambda_i \sum_{j=1}^{n} D_j(p(1 + \theta_j)), \]

which yields the following necessary conditions

\[ p \theta_i = -\lambda_i, \quad \dot{\lambda}_i = r \lambda_i, \quad (13) \]

which establish that the price is a *strategic substitute* of the tariff rate. Notice that now, as the extraction costs are supported directly by the monopolist, the tariff, \( p \theta_i \), must increase at the interest rate.

For the monopolist the Hamiltonian is

\[ H_M = (p - c(x) + \lambda_M) \sum_{i=1}^{n} D_i(p(1 + \theta_i)), \]

and the necessary conditions are

\[ \sum_{i=1}^{n} D_i + (p - c + \lambda_M) \sum_{i=1}^{n} (1 + \theta_i) D'_i = 0, \quad (14) \]

\[ \dot{\lambda}_M = r \lambda_M + c' \sum_{i=1}^{n} D_i, \quad (15) \]

where (14) is the *instantaneous reaction function* of the monopoly. By differentiation we can obtain that

\[ \frac{\partial p}{\partial \theta_i} = -\frac{pD'_i + (p - c + \lambda_M)(D'_i + p(1 + \theta_i) D''_i)}{2 \sum_{i=1}^{n} (1 + \theta_i) D'_i + (p - c + \lambda_M) \sum_{i=1}^{n}(1 + \theta_i)^2 D''_i} < 0, \quad (16) \]

and so we can establish that

**Proposition 3** If the demand functions are concave, \( D''_i \leq 0 \), the tariff rate of one importing country is a strategic substitute of the monopoly price.
With $D''_i \leq 0$, the numerator and the denominator of (16) are negative since by (14) $(p - c + \lambda_M)$ must be positive. Moreover, it is easy to check that (14) is the standard condition that characterizes the monopoly equilibrium: marginal revenue equal to marginal cost, now including the user cost of the resource. (14) can be written as

$$
\frac{\sum_{i=1}^{n} D_i}{\sum_{i=1}^{n} (1 + \theta_i) D'_i} + p = c - \lambda_M,
$$

and taking common factor $p$ as

$$
MR = p \left( \frac{1}{\varepsilon_{Q,p}} + 1 \right) = c - \lambda_M = MC,
$$

where $\varepsilon_{Q,p}$ is the elasticity of the aggregate demand function.

As in the previous game we have a system of $2n + 3$ equations to calculate the open-loop Nash equilibrium.

$$
p\theta_i = -\lambda_i, \quad i = 1, \ldots, n, \quad (17)
$$

$$
\dot{\lambda}_i = r\lambda_i, \quad i = 1, \ldots, n, \quad (18)
$$

$$
0 = \sum_{i=1}^{n} D_i + (p - c + \lambda_M) \sum_{i=1}^{n} (1 + \theta_i) D'_i, \quad (19)
$$

$$
\dot{\lambda}_M = r\lambda_M + c\sum_{i=1}^{n} D_i, \quad (20)
$$

$$
\dot{x} = \sum_{i=1}^{n} D_i. \quad (21)
$$

In order to calculate the steady state we have to take into account that the different countries can have different backstop prices. In this case, what is going to occur is that the countries are going to leave the market sequentially as the monopoly price reaches its backstop price so that the steady state will be defined by $\theta_i^\infty = \lambda_i^\infty = \lambda_M^\infty = 0, \quad i = 1, \ldots, n, \quad p^\infty = c(x^\infty)$ and $D_j(p^\infty) = 0$ where $j$ is the country with the highest backstop price.\footnote{We are aware that the nature of the game can change when the number of countries in the market is low because then the assumption of a price-taker representative consumer cannot work. In order to avoid this problem we assume that there are different types of countries with a very similar backstop price so that there are always enough countries in the market to support the assumption of a price-taker behaviour. Another possibility is to assume that in each country there are enough consumers to make the competitive assumption acceptable. This problem will not appear if the available backstop technology is the same for all countries.} Then it is straightforward that
**Proposition 4** The governments of the importing countries cannot capture the resource rent using a tariff, in other words, the open-loop Nash equilibrium tariff rate with depletion effects is zero.

This is an immediate consequence of the fact that the tariff, $p\theta_i$, must increase at a constant rate which is *incompatible* with the fact that ultimately the tariff rate must be zero because of the economic exhaustion of the resource. Notice that the price cannot be zero since at the steady state it is equal to the marginal extraction cost. Thus the unique path that can converge to the steady state requires that $\theta_i = \lambda_i = 0$ throughout the exploitation period of the resource and the system (17)-(18)-(19)-(20)-(21) yields the standard solution for the monopolistic extraction with depletion effects. As long as this argument does not depend on the cost structure, the result will also be valid when there are no depletion effects. It will be valid as well even if the importing countries cooperate imposing the same tariff rate on the resource importations, i.e., even if the importing country governments sign a *tariff agreement* and independently of whether we assume that they are symmetric or not. It is easy to show that the previous result also applies to the case of a *per unit tariff* since all the analysis for the per unit tariff is identical to the one developed in this section simply substituting $p\theta_i$ by the per unit tariff.

### 3.2 A Tariff Agreement: The Markov-perfect Nash Equilibrium

Next, we want to investigate whether this last result is a consequence of the equilibrium concept used to solve the game. To do so we propose in this section a game for which it is possible to calculate the stationary Markovian (feedback) strategies. Now we assume that the governments of the importing countries sign an agreement to impose the same *per unit tariff* on the resource importations with the aim of maximizing the discounted present value of the sum of the aggregate consumer’s welfare.\(^{10}\) In order to obtain an analytical solution for the game we also assume that the consumer’s gross surplus is given by $U_i(q_i) = aq_i - (1/2)q_i^2$ and that the extraction cost is linear, $c(x) = cx$. With these changes we have a differential game between a monopolist and a coalition of the importing country governments that can be written as follows for the monopolist,

\(^{10}\)As we have obtained the same qualitative results both for an *ad valorem* tariff and for a per unit tariff, the change in the specification of the tariff does not suppose a strong discontinuity in the analysis developed in this paper. Besides, a per unit tariff allows us to compute the Markov-perfect Nash equilibrium in linear strategies.
\[
\max_{\{p\}} \int_{0}^{\infty} e^{-rt} ((p - cx)n(a - p - \theta)) \, dt,
\]  
\tag{22}

and as follows for the coalition of the governments of the importing countries

\[
\max_{\{\theta\}} \int_{0}^{\infty} e^{-rt} n \left( (a - p)(a - p - \theta) - \frac{1}{2}(a - p - \theta)^2 \right) \, dt,
\]  
\tag{23}

the dynamic constraint being

\[s.t. \, \dot{x} = Q = n(a - p - \theta), \, x(0) = x_0 \geq 0.\]  
\tag{24}

Markov strategies must satisfy the following system of Hamilton-Jacobi-Bellman equations:

\[
r W_A = \max_{\{\theta\}} \left\{ n \left( (a - p)(a - p - \theta) - \frac{1}{2}(a - p - \theta)^2 \right) \right. \\
+ \left. W'_A n(a - p - \theta) \right\},
\]  
\tag{25}

\[
r W_M = \max_{\{p\}} \left\{ (p - cx)n(a - p - \theta) + W'_M n(a - p - \theta) \right\},
\]  
\tag{26}

where \(W_M(x)\) stands for the optimal current value functions associated with the dynamic optimization problem for the monopoly (22) and \(W_A(x)\) for the optimal current value functions associated with the dynamic optimization problem for the agreement (23); i.e., they denote the maxima of the objectives (22) and (23) subject to (24) for the current value of the state variable.

From the first-order conditions for the maximization of the right-hand sides of the HJB equations, we get the instantaneous reaction functions of the governments and the monopolist:

\[\theta = -W'_A,\]  
\tag{27}

\[p = \frac{1}{2} (a + cx - W'_M - \theta).\]  
\tag{28}

These expressions establish that the optimal tariff is independent of the monopoly price and equal, as before, to the user cost of the resource for the importing countries, and that the price and the tariff are strategic substitutes for the monopolist.

By substitution of (27) and (28), we get the solution of the price as a function of the first derivatives of the value functions: \(p = \frac{1}{2} (a + cx - W'_M + W'_A).\)
Next, by incorporating the optimal strategies into the HJB Eqs. (25) and (26), we eliminate the maximization and obtain, after some calculations, a pair of nonlinear differential equations:

\[ rW_A = \frac{n}{8} (a - cx + W'_A + W'_M)^2, \quad (29) \]

\[ rW_M = \frac{n}{4} (a - cx + W'_M + W'_A)^2. \quad (30) \]

In order to derive the solution to this system of differential equations, we guess quadratic representations for the value functions \( W_A \) and \( W_M \),

\[ W_A(x) = \frac{1}{2} \alpha_A x^2 + \beta_A x + \mu_A, \quad W_M(x) = \frac{1}{2} \alpha_M x^2 + \beta_M x + \mu_M, \quad (31) \]

and we apply the same procedure as the one used by Wirl and Dockner (1995) to calculate the coefficients, see Appendix A. Substituting these coefficients in (27) and (28), we obtain the linear Markov-perfect Nash equilibrium strategies for the tariff and the price:

\[ \theta = \frac{an\delta}{4r + 3n\delta} \frac{n\delta^2}{4r} x, \quad (32) \]

\[ p = \frac{2a(x + n\delta)}{4r + 3n\delta} + \frac{2c + \delta}{6} x, \quad (33) \]

where

\[ \delta = \frac{2}{3n} \left( (3cnr + r^2)^{0.5} - r \right) > 0. \]

By visual inspection it can be seen that the tariff is inversely related to the accumulated extractions whereas the price increases with the exploitation of the resource. Now using the equilibrium strategies, differential equation (24) can be solved, yielding

\[ x = (x_0 - \frac{a}{c}) \exp \left\{ -\frac{n\delta}{2} t \right\} + \frac{a}{c}, \quad (34) \]

and by substitution in the equilibrium strategies the tariff and price dynamics are obtained.\(^{11}\)

\[ \theta = \frac{an\delta^2}{4rc} \exp \left\{ -\frac{n\delta}{2} t \right\}, \quad p = a \left( 1 - \frac{2c + \delta}{6c} \exp \left\{ -\frac{n\delta}{2} t \right\} \right). \quad (35) \]

\(^{11}\)In order to simplify the presentation we assume that \( x_0 = 0 \). This does not change the sign of the dynamics of these two variables.
Finally, the consumer price can be simply calculated by the addition of the monopoly price and tariff.

\[ \pi = \theta + p = a \left( 1 - \frac{\delta}{2c} \exp \left\{ -\frac{n\delta t}{2} \right\} \right) \]  

so we can conclude that

**Proposition 5** The Markov-perfect Nash equilibrium tariff rate decreases throughout the exploitation period of the resource and converges to zero in the long run. Moreover, the monopoly and consumer prices are increasing and converge to the backstop price.

Clearly, these results establish that the commitment that the open-loop Nash equilibrium requires for the importing countries, a commitment for the entire exploitation period of the resource, drastically reduces the possibilities of using a tariff to capture part of the monopoly’s rents. In other words, the importing country governments have to play feedback strategies, i.e., to define the optimal policy as a function of the accumulated extractions, in order to be able to impose an advantageous tariff for the consumers.

### 4 The Case of a Quantity-setting Monopolist

Until now we have assumed that the monopoly chooses the price and the market establishes the resource extraction rate. In this section we analyze the other possibility the monopoly has: to choose the quantity and leave the market to set up the price. In this case by substitution of the inverse demand function in the instantaneous consumer’s welfare the following expression is obtained

\[ W_i = U_i \left( D_i \left( p(Q, \bar{\theta}) (1 + \theta_i) \right) \right) - p(Q, \bar{\theta})D_i \left( p(Q, \bar{\theta})(1 + \theta_i) \right), \]  

where \( \bar{\theta} \) is the vector of the tariff rates.

As the extraction rate is determined by the monopolist, the governments of the importing countries have no influence on the dynamics of the stock. For this reason, in this case the tariff rate is given by the Nash equilibrium of the static game defined by (37). In other words, at each point in time the importing countries choose a tariff rate to maximize the instantaneous flow of the consumer’s welfare given the extraction rate and the rival’s tariff rates.

The first order conditions yield

\[ (U'_i - p)D'_i \left( \frac{\partial p}{\partial \theta_i} (1 + \theta_i) + p \right) = \frac{\partial p}{\partial \theta_i} D_i, \quad i = 1, \ldots, n, \]
and as $U'_i = p(1 + \theta_i)$ is obtained

$$p\theta_i D'_i \left( \frac{\partial p}{\partial \theta_i}(1 + \theta_i) + p \right) = \frac{\partial p}{\partial \theta_i} D_i.$$  

By differentiation of the demand function we get

$$\frac{\partial p}{\partial \theta_i} = -\frac{D'_i p}{\sum_{j=1}^n D'_j (1 + \theta_j)}$$

that by substitution into the above expression yields

$$p\theta_i = -\frac{D_i}{\sum_{j\neq i} D'_j (1 + \theta_j)}, \quad i = 1, ..., n. \quad (38)$$

This is the version for an “ad valorem” tariff of the one obtained by Karp and Newbery (1991, p. 288) for a per unit tariff.

Assuming that the system (38) has a unique solution, the dynamic of the extraction rate and hence the dynamics of the tariff rate can be calculated as the solution of a standard optimal control problem.\(^{12}\) This result shows that the nature of the game changes when the monopoly sets the quantity instead of the price. Now at each moment, given the extraction rate, the importing countries can use the tariffs to reduce the monopoly price and in this way, increase domestic welfare.

### 4.1 A Tariff Agreement: The Markov-Perfect Stackelberg Equilibrium

In order to complete the analysis of the previous section we look now at the game between the monopolist and the coalition of the importing countries.

When the monopolist chooses the extraction rate, the monopoly price depends on the tariff selected by the countries in the tariff agreement according to the demand inverse function

$$p = a - \theta - (Q/n), \quad (39)$$

so that the instantaneous aggregate welfare of the importing country consumers is written as

$$W_A = n \left( a \frac{Q}{n} - \frac{1}{2} \left( \frac{Q}{n} \right)^2 - \left( a - \theta \frac{Q}{n} \right) \frac{Q}{n} \right).$$

\(^{12}\)For a linear demand the existence of a solution could be shown at least for the symmetric case although not the uniqueness. For a per unit tariff both the existence and the uniqueness can be shown.
From this expression it is pretty obvious that the optimal policy is to choose, for a given quantity, a tariff such that the price be zero: \( \theta = a - (Q/n) \). However, in this case, the monopoly has no interest in exploiting the resource so that finally in the open-loop Nash equilibrium of the game, the importing countries are not going to obtain any surplus. Given this result and the structure of the game, we think that a Stackelberg equilibrium better represents the relationship between the countries in the tariff agreement and the monopolist. What happens is that the influence of the tariff rate on the monopoly price gives a strategic advantage to the countries in the tariff agreement so that they can behave as a leader.\(^{13}\) Next, we show that the importing countries are interested in establishing a tariff that gives the monopolist the possibility of obtaining a positive price with the aim of obtaining a positive surplus for their consumers. A policy that is clearly superior to the one established above.

Since it is well known that an open-loop Stackelberg equilibrium besides not being subgame perfect (strong time inconsistency) can also be temporally inconsistent (weak time inconsistency), we propose in this section to calculate a Markov-perfect Stackelberg equilibrium which will satisfy the weak time consistency as well. The method of obtaining a Markov-perfect Stackelberg equilibrium we use in this paper was first proposed by Simaan and Cruz (1973). The method is for the leader to treat the follower’s HJB equation as a constraint, and to solve his own problem using dynamic programming.

In order to calculate this kind of equilibrium we need the instantaneous reaction function of the follower, i.e., of the monopolist, which is obtained from the following HJB equation where the price is given by (39)

\[
rW_M = \max \{Q \} \{(a - \theta - (Q/n) - cx)Q + W'_M Q\}.
\]

The first-order condition for the maximization of the right-hand side of this equation yields

\[
Q = \frac{n}{2} (a - \theta - cx + W'_M), \tag{40}
\]

the monopoly’s instantaneous reaction function.

Then the HJB equation for the coalition of the importing country gov-

\(^{13}\)Lewis, Lindsey and Ware (1986) have analyzed the interaction between a resource monopolist and a coalition of consumers that act collectively to introduce a durable long-lived substitute. They compare the equilibrium predictions of a non-commitment model with two other models where the monopolist and the resource consumer act as time-committed Stackelberg leaders.
ernments can be written as
\begin{align*}
rW_A &= \max_{\theta} \left\{ \frac{n}{8} \left( (a - cx + W_M')^2 + 2(a - cx + W_M') \theta - 3\theta^2 \right) \\
&\quad + W_A' \frac{n}{2} (a - \theta - cx + W_M') \right\}, \quad (41)
\end{align*}
where (39) and (40) have been used to calculate the monopoly price and resource importations \( q = Q/n \). The first-order condition for the maximization of the right-hand side of this equation yields the optimal policy or strategy for the tariff which allows us to calculate the optimal policy for the price using (40)
\begin{align*}
\theta &= \frac{1}{3} (a - cx - 2W_A' + W_M'), \\
Q &= \frac{n}{3} (a - cx + W_A' + W_M'). \quad (42, 43)
\end{align*}
By substitution of the optimal tariff into the HJB equation of the importing country governments and of the tariff and extraction rates into the HJB equation of the monopoly, we eliminate the maximization and obtain, after some manipulations, the following pair of nonlinear differential equations
\begin{align*}
rW_A &= \frac{n}{6} (a - cx + W_A' + W_M')^2, \quad (44) \\
rW_M &= \frac{n}{9} (a - cx + W_A' + W_M')^2. \quad (45)
\end{align*}
Now, proceeding in the same way as in the previous section, we get the linear Markov-perfect Stackelberg equilibrium strategies for the tariff and extraction rates
\begin{align*}
\theta &= \frac{3a(r + n\gamma)}{9r + 5n\gamma} - \frac{9cr + 4n\gamma^2}{27r} x, \\
Q &= \frac{3an\gamma}{9r + 5n\gamma} - \frac{n\gamma}{3} x, \quad (46, 47)
\end{align*}
where
\[ \gamma = \frac{3}{10n} \left( (20cnr + 9r^2)^{0.5} - 3r \right) > 0. \]
By visual inspection it can be seen that the tariff and extraction rates are inversely related to the accumulated extractions.
Since \( \dot{x} = Q \), we can use (47) to calculate the dynamics of the accumulated extractions for \( x_0 = 0 \)
\begin{align*}
x &= \frac{a}{c} \left( 1 - \exp \left\{ -\frac{\gamma}{3} t \right\} \right), \quad (48)
\end{align*}
and by substitution in the equilibrium strategies the dynamics of the tariff and extraction rates

\[
\theta = \frac{a(9cr + 4n\gamma^2)}{27cr} \exp \left\{ -\frac{n\gamma}{3} t \right\}, \quad Q = \frac{an\gamma}{3c} \exp \left\{ -\frac{n\gamma}{3} t \right\}.
\]

(49)

Now by substitution in the demand inverse function the monopoly price can be calculated,

\[
p = a \left( 1 - \frac{9cr + 9r\gamma + 4n\gamma^2}{27cr} \exp \left\{ -\frac{n\gamma}{3} t \right\} \right),
\]

(50)

and adding this price to the tariff rate, the consumer price is

\[
\pi = \theta + p = a \left( 1 - \frac{\gamma}{3c} \exp \left\{ -\frac{n\gamma}{3} t \right\} \right),
\]

(51)

so we can conclude that

**Proposition 6** The Markov-perfect Stackelberg equilibrium tariff and extraction rates decrease throughout the exploitation period of the resource and converge to zero in the long run. Moreover, the monopoly and consumer prices are increasing and converge to the backstop price.

This result along with the previous one, Prop. 5, establish that it is advantageous for the importing countries to coordinate and impose a common tariff on the resource importations, both if the monopoly chooses the price and if the monopoly chooses the extraction rate. However, in this second case, the importing countries enjoy a strategic advantage and can impose a higher tariff rate as we show in the next section.

## 5 Comparing the Two Equilibria

This section compares the Nash equilibrium (MPNE) and the Stackelberg equilibrium in which importing countries act as a leader (MPSE). First, we compare the initial values of the optimal strategies.

**Lemma 1** The initial consumer price and tariff rate are lower and the initial monopoly price is higher in the MPNE than in the MPSE.
Proof. See Appendix B. □

This result establishes that the strategic advantage of the importing country governments translates into a higher initial value for the tariff, which reduces the demand for the resource. The reduction in initial demand explains why the initial monopoly price is lower in the MPSE. Thus, a higher tariff has two effects on the consumer price: one direct and positive and another indirect and negative through the monopoly price. The net effect is positive because the reduction in demand does not completely translate into the monopoly price, given that the demand function is linear and the marginal extraction cost is constant. Hence, the initial consumer price is lower in the MPNE although the monopoly price is higher.

We now turn to the transitional dynamics.

Proposition 7 The tariff in the MPSE is above the MPNE tariff. Contrarily, the monopoly price in the MPSE is below the MPNE monopoly price. However, the consumer price in the MPSE is first above, but later below, the MPNE consumer price.

Proof. See Appendix C. □

This result is a logical consequence of the fact that both equilibria converge to the same long run equilibrium characterized by the economic exhaustion of the resource. Accordingly, the total amount mined is the same – irrespective of the equilibrium concept used to solve the game - and the area under the temporal path of the extraction rate must therefore be the same as well. The temporal paths must thus intersect. The monotonic behaviour of the variables explains why the paths intersect only once. The intersection of the temporal paths of the extraction rate occurs along with the intersection of the temporal paths of the consumer price. However, for the tariff and monopoly price there are no intersection points. This is possible because of the inverse relationship between the tariff and monopoly price for both equilibria. In the MPSE the tariff is higher than the tariff in the MPNE whereas the monopoly price is lower. Then as the consumer price is given by the tariff plus the monopoly price, the consumer price can be first higher, and later lower, in the MPSE than in the MPNE.

Moreover, it is easy to show that although the leadership position is advantageous for the importing countries, the efficiency of the market decreases. This is a standard result in the comparison between the Nash and Stackelberg equilibria that we do not show here.14

14It is easy to show for $x_0 = 0$ that when the importing country governments have a strategic advantage, the aggregate consumer’s welfare increases while the monopoly’s rent and aggregate welfare (measured as the aggregate consumer’s welfare plus monopoly’s
6 Conclusions

In this paper we have revisited the issue, first tackled by Bergstrom (1982), of using a tariff on a nonrenewable resource importations in order to appropriate part of the resource rents. We extend the analysis taking into account that the exploitation of nonrenewable resources is characterized by the presence of depletion effects, i.e., the marginal extraction cost increases for the same extraction rate as the accumulated extractions increase. For the case of a competitive market we find that the optimal policy is a constant tariff if extraction is costless. This result supports Bergstrom’s finding. However, with depletion effects, the optimal tariff must be ultimately decreasing.

For the case of a monopolistic market the results depend crucially on the kind of strategies the importing country governments play and on whether the monopolist chooses the price or the extraction rate. For a price-setting monopolist we show that the importing countries cannot use a tariff to capture the monopoly rents if they are constrained to use open-loop strategies, i.e., if they commit to a temporal path for the tariff, even if the governments sign a tariff agreement to impose the same tariff. This result drastically changes if the importing countries co-operate through a tariff agreement and use Markov (feedback) strategies, i.e., if they commit to a rule that fixes the tariff as a function of the accumulated extractions (the state variable of the game). In this case a tariff is clearly advantageous for the consumers of the importing countries. For a quantity-setting monopolist the nature of the game changes, in fact, for the importing country governments the game becomes a static game and now the importing countries find it advantageous to set a tariff on resource importations. Finally, we show that when the governments of the importing countries sign a tariff agreement they enjoy a strategic advantage which allows them to act as the leader of the game.

Although we think that this paper clarifies and extends the analysis of the possibilities of using a tariff to capture nonrenewable resource rents it would be of interest to address this issue when there is no cooperation among the importing country governments for the case of a price-setting monopolist. In particular, we have calculated the Markov-perfect Nash equilibrium for a differential game between a coalition of importing country governments and a monopolist that sets the price but, although we guess that the qualitative result is not going to change, it would be useful to know whether the importing countries can gain by imposing a feedback tariff without coordination.

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rent) decrease, compared with the MPNE. Notice that for $x_0 = 0$ the comparison between the value functions of the importing countries and the monopoly reduces to the comparison of the independent term of the quadratic representations. See, for instance, the welfare analysis developed in Rubio and Escriche (2001).
i.e., without signing a tariff agreement.

## A Derivation of the Stationary Linear Markov Strategies

Substituting \( W_A, W'_A, W_M \) and \( W'_M \) into (29) and (30), collecting terms with equal powers of \( x \), and equating the coefficients of these terms to zero, one obtains the following system of coupled Riccati equations

\[
4r\alpha_A = n(c - \alpha_A - \alpha_M)^2, \quad (52)
\]
\[
2r\alpha_M = n(c - \alpha_A - \alpha_M)^2, \quad (53)
\]
\[
4r\beta_A = -n(a + \beta_A + \beta_M)(c - \alpha_A - \alpha_M), \quad (54)
\]
\[
2r\beta_M = -n(a + \beta_A + \beta_M)(c - \alpha_A - \alpha_M). \quad (55)
\]

These equations can be explicitly solved by the following change in the variables: \( y = \alpha_A + \alpha_M \) and \( z = \beta_A + \beta_M \). Adding the first two equations and the last two equations yields a system in the new variables

\[
4ry = 3n(c - y)^2, \quad (56)
\]
\[
4rz = -3n(a + z)(c - y). \quad (57)
\]

The solution for the first equation is

\[
y = c + 2r \frac{2}{3n} \pm \frac{2}{3n} \left(3cnr + r^2\right)^{0.5}. \quad (58)
\]

In order to choose between the two roots a stability condition is used. Next, we develop this stability condition. Using the proposed value functions the linear Markov strategies can be written as

\[
\theta = -\alpha_A x - \beta_A, \quad p = \frac{1}{2} (a + \beta_A - \beta_M + (c + \alpha_A - \alpha_M)x), \quad (59)
\]

so that the dynamics of the accumulated extractions is given by

\[
\dot{x} = \frac{n}{2} (a + \beta_A + \beta_M - (c - \alpha_A - \alpha_M)x).
\]

Then, we have that

\[
\frac{d\dot{x}}{dx} < 0 \rightarrow \frac{d\dot{x}}{dx} = -\frac{n}{2} (c - \alpha_A - \alpha_M) = -\frac{n}{2} (c - y) < 0,
\]

24
so that the stability condition requires that $c - y > 0$. This condition is satisfied by the lowest root of (58) yielding

$$
\delta = c - y = \frac{2}{3n} \left( (3cnr + r^2)^{0.5} - r \right) > 0.
$$

(60)

With this result $\alpha_A$ and $\alpha_M$ can be obtained directly from (52) and (53)

$$
\alpha_A = \frac{\delta^2}{4r}, \quad \alpha_M = \frac{\delta^2}{2r}.
$$

(61)

Next, we calculate $z$ using (57)

$$
z = -\frac{3an\delta}{4r + 3n\delta} < 0,
$$

and then $\beta_A$ and $\beta_M$ from (54) and (55)

$$
\beta_A = -\frac{an\delta}{4r + 3n\delta} < 0, \quad \beta_M = -\frac{2an\delta}{4r + 3n\delta} < 0.
$$

(62)

Finally, by substitution in (59) we obtain the linear Makov-perfect Nash equilibrium strategies for the tariff and the price (32) and (33).

### B Proof of Lemma 1

Let us suppose that $\theta^S(0) \leq \theta^N(0)$.\textsuperscript{15} Then using (35) and (49) for $t = 0$ we obtain after obvious simplifications that $36cr + 16n\gamma^2 \leq 27n\delta^2$. In Appendix A we have established that $4ry^N = 3n(c - y^N)^2 = 3n\delta^2$, see (56). On the other hand, the Riccati equations for the MPSE yield $9ry^S = 5n(c - y^S)^2 = 5n\gamma^2$ where $\gamma = c - y^S$ by definition. Then by substitution of $n\gamma^2$ and $n\delta^2$ in the above inequality we obtain that $5\delta + y^S \leq 0$. Developing $9ry^S = 5n(c - y^S)^2$ we obtain the following quadratic equation $(y^S)^2 - (2c + (9r/5n))y^S + c^2 = 0$ which has two positive roots, the lowest root being the one that satisfies the stability condition so that $\gamma = c - y^S > 0$. Then as $\delta$ is positive, see (60) and $y^S$ as well, we have gotten a contradiction $5\delta + y^S \leq 0$, and $\theta^S(0) > \theta^N(0)$ is established.

Next, we compare the initial monopoly prices. Let us suppose that $p^S(0) \geq p^N(0)$. Then using (35) and (50) for $t = 0$ we obtain after obvious simplifications that $54r\gamma + 24n\gamma^2 \leq 27r\delta$. Using again that $9ry^S = 5n\gamma^2$ we obtain after substituting $n\gamma^2$ in the previous inequality and rearranging

\textsuperscript{15}Superscript $N$ stands for the MPNE and $S$ for the MPSE.
terms that $81c + 54\gamma + 135y^N \leq 0$. Where $y^N$ is the lowest root of (58). It is very easy to show that this root is positive so that a contradiction is established since $\gamma$ is also positive. Then we have that $p^S(0) < p^N(0)$. Finally, we compare the initial consumer prices. Using (36) and (51) we have that

$$\pi^S(0) - \pi^N(0) = \frac{a}{c} \left( \frac{\delta}{2} - \frac{\gamma}{3} \right),$$

which yields by substitution of $\delta$ and $\gamma$

$$\pi^S(0) - \pi^N(0) = \frac{a}{c} \left( \frac{c}{6} + \frac{y^S}{3} - \frac{y^N}{2} \right),$$

and now by substitution of $y^S$ and $y^N$

$$\pi^S(0) - \pi^N(0) = \frac{a}{c} \left( \frac{1}{3n} (3cnr + r^2)^{0.5} - \frac{r}{30n} - \frac{1}{10n} (20cnr + 9r^2)^{0.5} \right).$$

Let us suppose that this difference is negative or zero. Then we can write reordering terms and simplifying

$$10(3cnr + r^2)^{0.5} \leq r + 3(20cnr + 9r^2)^{0.5}.$$

Squaring, reordering terms and squaring again we have the contradiction: $4cn + r \leq 0$. Thus, we obtain that $\pi^S(0) > \pi^N(0)$, which also implies that $\delta/2 - \gamma/3$ is positive.

C Proof of Proposition 7

For the comparison of the tariff temporal paths, we use (35) and (49). The difference between the two temporal paths is given by

$$\theta^S - \theta^N = \theta^S(0) \exp \left\{ - \frac{n\gamma}{3} t \right\} - \theta^N(0) \exp \left\{ - \frac{n\delta}{2} t \right\}.$$

For $t = 0$ we know from Lemma 1 that the difference $\theta^S(0) - \theta^N(0)$ is positive. For $t \neq 0$ we can find the number of intersection points from the equation $\theta^S - \theta^N = 0$, which can be written as

$$\frac{\theta^N(0)}{\theta^S(0)} = \exp \left\{ n \left( \frac{\delta}{2} - \frac{\gamma}{3} \right) t \right\}.$$
However, this equation has no solution for \( t \geq 0 \) since the l.h.s. is a positive constant less than one and the r.h.s. is an increasing and convex function which takes the unit value for \( t = 0 \), and tends to infinity when \( t \) tends to infinity since as it has been shown in Lemma 1 \( \delta/2 - \gamma/3 \) is positive. Hence, the temporal path of the MPSE tariff is above the temporal path of the MPNE in the interval \([0, \infty)\). The same procedure can be used to show that the temporal path of the MPSE monopoly price is below the temporal path of the MPNE in the interval \([0, \infty)\). For comparing the temporal paths of the consumer price we calculate the difference between the two temporal paths using (36) and (51)

\[
\pi^S - \pi^N = \frac{a\delta}{2c} \exp \left\{ -\frac{n\delta}{2} t \right\} - \frac{a\gamma}{3c} \exp \left\{ -\frac{n\gamma}{3} t \right\},
\]

and we can find the number of intersection points from the equation \( \theta^S - \theta^N = 0 \) given by

\[
\frac{\delta/2}{\gamma/3} = \exp \left\{ n \left( \frac{\delta}{2} - \frac{\gamma}{3} \right) t \right\}, \tag{63}
\]

where the l.h.s. is a positive constant higher than one and the r.h.s. is an increasing and convex function which takes the unit value for \( t = 0 \), and tends to infinity when \( t \) tends to infinity as we have just seen. Hence, the temporal paths cut each other once in the interval \([0, \infty)\), and consequently, for \( 0 \leq t < t' \), where \( t' \) is the solution to Eq. (63), the MPSE consumer price is above the MPNE consumer price, whereas for \( t' < t \) the relationship is the contrary.

References


